MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210)

EME3026 - FLUID DYNAMICS

(ME)

06 FEBRUARY 2023
02.30 p.m. - 04.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of eight pages (including the cover page) with four questions and an Appendix.

2. Answer ALL four questions.

3. Each question carries 25 marks and the distribution of the marks for each question is given in brackets [ ].

4. Write all your answers in the answer booklet provided.

5. Property table booklet is provided for your reference.

Question 1

Converging-diverging nozzle is attached at the exhaust of a rocket engine to increase the outlet velocity as shown in the following figure. The combusted high speed exhaust gases enter the converging section at the velocity of 504 m/s. The pressure and temperature at the nozzle inlet are 756 kPa and 1680 K. The exhaust gases accelerate in the converging section and reach sonic state at the throat, where the throat area, $A_2 = A_* = 0.08\ \text{m}^2$. The gases continue to accelerate in the diverging section and the Mach number at the outlet, $M_3 = 6.6$. No normal shock wave is formed inside the nozzle.

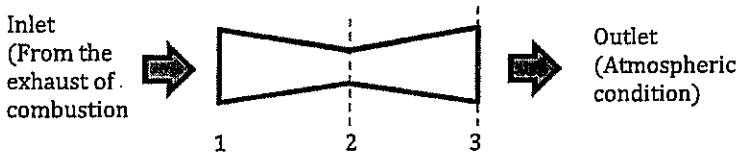

Figure Q1

Assuming ideal gas for exhaust gases where the specific heat ratio, $\gamma = 1.4$ and gas constant, $R = 300$ J/kg·K.

a. Find the Mach number at the inlet.

[3 marks]

b. Find the density at the inlet.

[3 marks]

c. Calculate the area of the inlet section.

[3 marks]

d. Find the mass flow rate of gases through the nozzle.

[3 marks]

e. Calculate the area of the outlet section.

[3 marks]

f. Determine the outlet temperature.

[3 marks]

g. Calculate the outlet velocity, density and pressure.

[7 marks]

Continued...

Question 2

Water is pumped from an elevated storage tank to a particular outlet through a vertical and horizontal pipe as shown in the following figure. The initially close valve is opened instantaneously and water starts to flow. The water flow velocity in the pipe increases with respect to time where the velocity can be expressed as the function of time, $u(t)$. The rigid pipe has 50 mm inner diameter and is very rough where the Darcy friction factor, $f = 0.01962$. The water head is 30 m and the minor losses (including the internal losses in the pump, elbow and valve) is 2 m.

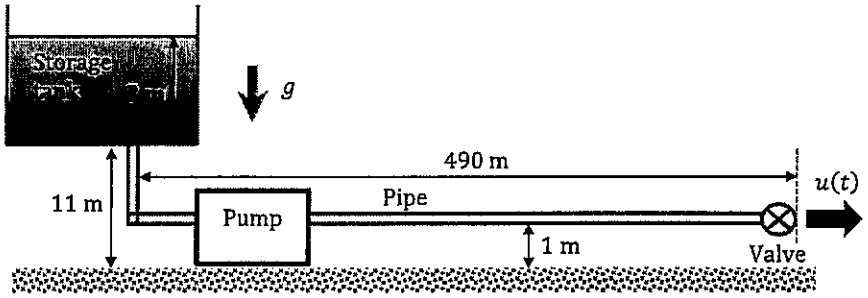

Figure Q2

Assume incompressible flow, and water density, $\rho = 1000$ kg/m^3 and gravitational acceleration, $g = 9.81$ m/s^2.

a. Derive the momentum equation of water flow in the horizontal pipe.

[8 marks]

b. Determine the time dependent velocity function, $u(t)$.

[8 marks]

c. Find the flow velocity, 2 seconds after the valve is opened.

[3 marks]

d. Determine the steady state velocity.

[3 marks]

e. Calculate the mass flow rate of water at steady state.

[3 marks]

Continued...

Question 3

The complex potential function of a potential flow, $\Phi(z) = Cz^4 = C(x + iy)^4$, where the constant, $C = 1$ cm^{-2}s^{-1}, and, x, and, y, are in cm. The density of fluid is 1.2 kg/m^3 and gravity is acting in opposite direction parallel to the $y -$ axis.

a. Determine the corresponding stream function and potential function in both Cartesian and polar coordinates.

[8 marks]

b. Locate the stagnation point in the flow field.

[3 marks]

c. At point A, $x = 4, y = 3$, find the magnitude of velocity, equation of streamline and potential lines.

[7 marks]

d. Calculate the pressure difference between point A and the origin.

[7 marks]

Continued...

Question 4

Consider a steady air flow of velocity, U, (parallel to the x −axis) over a flat delta wing with the length of 1 m and span of 4 m as shown in the following figure.

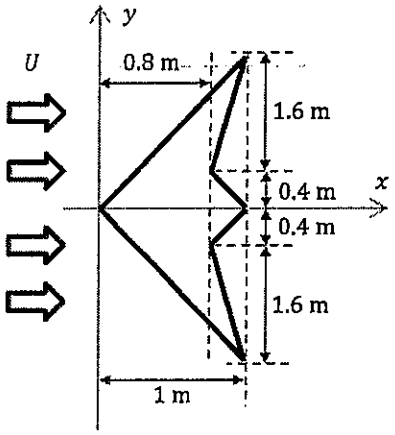

Figure Q4

The skin friction coefficient of flat plate can be expressed in term of Reynolds number, Re_x, for laminar, and turbulent flow as, $C_f = 0.664(Re_x)^{-\frac{1}{2}}$, for, $Re_x \leq 5 \times 10^5$, and, $C_f = 0.02708(Re_x)^{-\frac{1}{7}}$, for, $Re_x > 5 \times 10^5$, respectively. Assume air density, $\rho = 1.23$ kg/m^3 and kinematic viscosity, $\nu = 1.5 \times 10^{-5}$ m^2/s.

a. If $U = 3$ m/s, calculate the corresponding Reynolds number of the flow.
[2 marks]

b. For $U = 3$ m/s, determine the drag force applied on the delta wings (include upper and lower surface).
[9 marks]

c. If $U = 12.5$ m/s, calculate the corresponding Reynolds number of the flow.
[2 marks]

d. For $U = 12.5$ m/s, find the transition length, which measure the distance from the leading edge.
[2 marks]

e. For $U = 12.5$ m/s, determine the drag force applied on the delta wings (include upper and lower surface).
[10 marks]

Continued...

APPENDIX A : SUMMARY OF FORMULAS

A1. Governing equations of incompressible flow in Cartesian coordinates:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=g_x-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=g_y-\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right)+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=g_z-\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right)+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

A2. Governing equations of incompressible flow in cylindrical coordinates:

$$\frac{1}{r}\left[\frac{\partial(ru_r)}{\partial r}\right]+\frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta}\right)+\frac{\partial u_z}{\partial z}=0$$

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\left(\frac{\partial u_r}{\partial \theta}\right)+u_z\frac{\partial u_r}{\partial z}-\frac{{u_\theta}^2}{r}=g_r-\frac{1}{\rho}\left(\frac{\partial p}{\partial r}\right)+\nu\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\left(\frac{\partial u_r}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2 u_r}{\partial \theta^2}\right)+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}-\frac{2}{r^2}\left(\frac{\partial u_\theta}{\partial \theta}\right)\right]$$

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\left(\frac{\partial u_\theta}{\partial \theta}\right)+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}=g_\theta-\frac{1}{\rho r}\left(\frac{\partial p}{\partial \theta}\right)+\nu\left[\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1}{r}\left(\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2 u_\theta}{\partial \theta^2}\right)+\frac{\partial^2 u_\theta}{\partial z^2}-\frac{u_\theta}{r^2}+\frac{2}{r^2}\left(\frac{\partial u_r}{\partial \theta}\right)\right]$$

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\left(\frac{\partial u_z}{\partial \theta}\right)+u_z\frac{\partial u_z}{\partial z}=g_z-\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right)+\nu\left[\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\left(\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2 u_z}{\partial \theta^2}\right)+\frac{\partial^2 u_z}{\partial z^2}\right]$$

A3. Continuity, linear momentum, energy, vorticity, and Bernoulli equations of incompressible flow in vector form:

$$\nabla\cdot\mathbf{u}=0$$

$$\rho\left[\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}\right]=\rho\mathbf{g}-\nabla p+\mu(\nabla\cdot\nabla)\mathbf{u}$$

$$\rho\frac{\partial e_t}{\partial t}+\rho(\mathbf{u}\cdot\nabla)e_t-\nabla\cdot(k\nabla T)-\nabla\cdot(\boldsymbol{\sigma}_{ij}\mathbf{u})=0$$

$$\frac{\partial \boldsymbol{\Omega}}{\partial t}+(\mathbf{u}\cdot\nabla)\boldsymbol{\Omega}-(\boldsymbol{\Omega}\cdot\nabla)\mathbf{u}+\nu(\nabla\cdot\nabla)\boldsymbol{\Omega}=0$$

$$\frac{\partial}{\partial t}\left(\frac{\|\mathbf{u}\|^2}{2}\right)+\mathbf{u}\cdot\nabla\left(\frac{1}{2}\|\mathbf{u}\|^2+\frac{p}{\rho}-G\right)=0$$

A4. Stream functions and potential function in Cartesian and cylindrical coordinates:

$$\mathbf{u}\cdot\nabla\psi=0\,;\ u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x}\,;\ u_r=\frac{1}{r}\left(\frac{\partial\psi}{\partial\theta}\right),u_\theta=-\frac{\partial\psi}{\partial r}$$

$$\mathbf{u}=\nabla\phi\,;\ u=\frac{\partial\phi}{\partial x},v=\frac{\partial\phi}{\partial y}\,;\ u_r=\frac{\partial\phi}{\partial r},u_\theta=\frac{1}{r}\left(\frac{\partial\phi}{\partial\theta}\right)$$

A5. Complex potential functions for corner flow, uniform flow, source/sink, vortex, and doublet:

$$\Phi'(z)=u-iv=(u_r-iu_\theta)e^{-i\theta}$$

$$\Phi(z)=Az^n$$

$$\Phi(z)=Ue^{-i\alpha}z=U(r\cos\theta\cos\alpha+r\sin\theta\sin\alpha)+i[U(r\sin\theta\cos\alpha-r\cos\theta\sin\alpha)]$$

$$\Phi(z)=\left(\frac{\dot{V}}{2\pi L}\right)\ln z=\left(\frac{\dot{V}}{2\pi L}\right)\ln r+i\left[\left(\frac{\dot{V}}{2\pi L}\right)\theta\right]$$

$$\Phi(z)=-i\left(\frac{\Gamma}{2\pi}\right)\ln z=\left(\frac{\Gamma}{2\pi}\right)\theta+i\left[-\left(\frac{\Gamma}{2\pi}\right)\ln r\right]$$

$$\Phi(z)=\frac{K}{z}=\frac{K\cos\theta}{r}+i\left[-\frac{K\sin\theta}{r}\right]$$

A6. Governing equations of incompressible boundary layer flow in Cartesian coordinates:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,;\ u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\left(\frac{dp}{dx}\right)+\nu\left(\frac{\partial^2 u}{\partial y^2}\right)\,;\ 0=-\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right)$$

A7. Momentum integral equation, boundary layer thickness, displacement thickness, and momentum thickness:

$$\frac{d\delta_m}{dx}+\left(\frac{2\delta_m+\delta_V}{U}\right)\left(\frac{dU}{dx}\right)=\frac{\tau_w}{\rho U^2}$$

$$u^*=u/U\,;\ y^*=y/\delta$$

$$C_V=\frac{\delta_V}{\delta}=\int_0^1(1-u^*)\,dy^*\,;\ C_m=\frac{\delta_m}{\delta}=\int_0^1\left(u^*-u^{*2}\right)dy^*\,;\ C_\tau=\left(\frac{du^*}{dy^*}\right)_{y^*=0}$$

$$\frac{\delta}{x}=\frac{\sqrt{2C_\tau}}{\sqrt{C_m}\sqrt{Re_x}}\,;\ \frac{\delta_V}{x}=\frac{C_V\delta}{x}=\frac{C_V\sqrt{2C_\tau}}{\sqrt{C_m}\sqrt{Re_x}}\,;\ \frac{\delta_m}{x}=\frac{C_m\delta}{x}=\frac{\sqrt{2C_mC_\tau}}{\sqrt{Re_x}}$$

A8. Blasius solution for laminar flow over a flat plate:

$f''' + 0.5ff'' = 0 ; f'(0) = 0 ; f(0) = 0 ; \lim_{\eta\to\infty} f'(\eta) = 1$

$\psi(x,y) = \sqrt{\nu U x} f(\eta) ; \eta(x,y) = y/\sqrt{\nu x/U}$

η	$f(\eta)$	$f'(\eta)$	$f''(\eta)$	η	$f(\eta)$	$f'(\eta)$	$f''(\eta)$	η	$f(\eta)$	$f'(\eta)$	$f''(\eta)$	η	$f(\eta)$	$f'(\eta)$	$f''(\eta)$
0.2	0.00664	0.06641	0.33198	2.2	0.78119	0.68131	0.24835	4.2	2.49804	0.96696	0.05052	6.2	4.47946	0.99936	0.00155
0.4	0.02656	0.13276	0.33147	2.4	0.92229	0.72898	0.22809	4.4	2.69236	0.97587	0.03897	6.4	4.67936	0.99961	0.00098
0.6	0.05973	0.19894	0.33008	2.6	1.07251	0.77245	0.20645	4.6	2.88825	0.98268	0.02948	6.6	4.87929	0.99977	0.00061
0.8	0.10611	0.26471	0.32739	2.8	1.23098	0.81151	0.18401	4.8	3.08532	0.98779	0.02187	6.8	5.07926	0.99986	0.00037
1.0	0.16557	0.32978	0.32301	3.0	1.39681	0.84604	0.16136	5.0	3.28327	0.99154	0.01591	7.0	5.27924	0.99992	0.00022
1.2	0.23795	0.39378	0.31659	3.2	1.56909	0.87608	0.13913	5.2	3.48187	0.99425	0.01134	7.2	5.47923	0.99996	0.00013
1.4	0.32298	0.45626	0.30787	3.4	1.74695	0.90176	0.11788	5.4	3.68092	0.99616	0.00793	7.4	5.67922	0.99998	0.00007
1.6	0.42032	0.51676	0.29666	3.6	1.92952	0.92333	0.09809	5.6	3.88029	0.99748	0.00543	7.6	5.87922	0.99999	0.00004
1.8	0.52952	0.57476	0.28293	3.8	2.11603	0.94112	0.08013	5.8	4.07988	0.99838	0.00365	7.8	6.07921	0.99999	0.00002
2.0	0.65002	0.62977	0.26675	4.0	2.30575	0.95552	0.06423	6.0	4.27962	0.99897	0.00240	8.0	6.27921	1.00000	0.00001

$\frac{\delta}{x} = \frac{4.919}{\sqrt{Re_x}} ; \frac{\delta_v}{x} = \frac{1.721}{\sqrt{Re_x}} ; \frac{\delta_m}{x} = \frac{0.664}{\sqrt{Re_x}} ; C_f = \frac{\tau_w}{\rho U^2/2} = \frac{0.664}{\sqrt{Re_x}}$

A9. Kolmogorov's length, time, and velocity scale:

$\eta = (\nu^3/\varepsilon)^{\frac{1}{4}} ; t_\eta = (\nu/\varepsilon)^{\frac{1}{2}} ; u_\eta = (\nu\varepsilon)^{\frac{1}{4}}$

A10. Root mean square of the turbulence fluctuation and intensity:

$u'_{rms} = \sqrt{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}} ; u'_{inten} = \frac{u'_{rms}}{\|\bar{u}\|} = \frac{\sqrt{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}}{\sqrt{\bar{u}^2 + \bar{v}^2 + \bar{w}^2}}$

$\overline{u'^2} = \frac{1}{N}\sum_{i=1}^{N} {u'_i}^2 \; ; \; \overline{v'^2} = \frac{1}{N}\sum_{i=1}^{N} {v'_i}^2 \; ; \; \overline{w'^2} = \frac{1}{N}\sum_{i=1}^{N} {w'_i}^2$

A11. Reynolds Average Navier Stokes equations for incompressible flow:

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho}\left(\frac{\partial \bar{p}}{\partial x}\right) + g_x + \frac{\partial}{\partial x}\left(\nu\frac{\partial \bar{u}}{\partial x} - \overline{u'^2}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial \bar{u}}{\partial y} - \overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial \bar{u}}{\partial z} - \overline{u'w'}\right)$$

$$\bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} + \bar{w}\frac{\partial \bar{v}}{\partial z} = -\frac{1}{\rho}\left(\frac{\partial \bar{p}}{\partial y}\right) + g_x + \frac{\partial}{\partial x}\left(\nu\frac{\partial \bar{u}}{\partial x} - \overline{u'v'}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial \bar{u}}{\partial y} - \overline{v'^2}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\right)$$

$$\bar{u}\frac{\partial \bar{w}}{\partial x} + \bar{v}\frac{\partial \bar{w}}{\partial y} + \bar{w}\frac{\partial \bar{w}}{\partial z} = -\frac{1}{\rho}\left(\frac{\partial \bar{p}}{\partial y}\right) + g_x + \frac{\partial}{\partial x}\left(\nu\frac{\partial \bar{u}}{\partial x} - \overline{u'w'}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial \bar{u}}{\partial y} - \overline{v'w'}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial \bar{u}}{\partial z} - \overline{w'^2}\right)$$

A12. Universal law of wall and velocity profile in turbulent layer:

$u^+ = \bar{u}/v_* ; y^+ = y/(\nu/v_*) ; v_* = \sqrt{\tau_w/\rho} ; y^* = y/y_0 ; \kappa = 0.41, C_1 = 5.0 ; \kappa = 0.40, C_1 = 5.5$

Viscous layer	$y^+ < 5$	$u^+ = y^+$
Buffer layer	$5 < y^+ < 40$	$y^+ = u^+ + e^{-\kappa B}\left[e^{-\kappa u^+} - 1 - \kappa u^+ - (\kappa u^+)^2/2 - (\kappa u^+)^3/6\right]$
Overlapping layer	$40 < y^+ < 350$	$u^+ = \kappa^{-1}\ln y^+ + C_1$
Outer layer	$y^+ > 350$	$u^+ = \kappa^{-1}\ln y^+ + C_1 + f_3(y^*)$

A13. Friction factor for laminar and turbulent pipe flow:

$u^+ = \bar{u}/v_* ; r^+ = (r_0 - r)/(\nu/v_*) ; {r_0}^+ = r_0/(\nu/v_*)$

Laminar	$Re_D < 2000$	$u^+ = \frac{r^+}{4} - \frac{(r^+)^2}{8{r_0}^+}$	$f = \frac{64}{Re_D}$	
Turbulent smooth	$Re_D \gg 2000$	$u^+ = (r^+)^{\frac{1}{7}}$	$f = 0.316(Re_D)^{-\frac{1}{4}}$	Blasius
Turbulent implicit		$u^+ = \frac{1}{0.41}\ln r^+ + 5.0$	$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{\xi/D}{3.7} + \frac{2.51}{(Re_D)\sqrt{f}}\right]$	Moody
Turbulent explicit		-	$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\xi/D}{3.7}\right)^{1.1} + \frac{6.9}{Re_D}\right]$	Haaland

A14. Skin friction coefficient for laminar and turbulent flow over a flat plate:

$u^+ = \bar{u}/v_* ; y^+ = y/(\nu/v_*)$

Laminar	$Re_L < 5\times10^5$	-	$C_f = 0.644(Re_x)^{-\frac{1}{2}}$	$C_f = 3.265(Re_\delta)^{-1}$
Turbulent smooth	$Re_L \gg 5\times10^5$	$u^+ = (y^+)^{\frac{1}{7}}$	$C_f = 0.05767(Re_x)^{-\frac{1}{5}}$	$C_f = 0.045(Re_\delta)^{-\frac{1}{4}}$
Turbulent smooth	$Re_L \gg 5\times10^5$	$u^+ = (y^+)^{\frac{1}{7}}$	$C_f = 0.02708(Re_x)^{-\frac{1}{7}}$	$C_f = 0.02(Re_\delta)^{-\frac{1}{6}}$
Turbulent rough	$Re_L \gg 5\times10^5$	-	$C_f = [2.87 + 1.58\log(x/\xi)]^{-2.5}$	-

A15. Ideal gas and constant specific heats relations:

$$p = \rho RT \; ; \; c_p - c_v = R \; ; \; ds = c_p\left(\frac{dT}{T}\right) - R\left(\frac{dp}{p}\right) \; ; \; c_p = \frac{\gamma R}{\gamma - 1} \; ; \; c_v = \frac{R}{\gamma - 1}$$

A16. Speed of sound:

$$a = \sqrt{(dp/d\rho)_s} \; ; \; a = \sqrt{\gamma RT} \text{ (ideal gas)}$$

A17. General equations of one-dimensional compressible flow:

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) = 0$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) = -\frac{\partial p}{\partial x}$$

$$\frac{\partial}{\partial t}(\rho h_t) + \frac{\partial}{\partial x}(\rho u h_t) = -\frac{\partial p}{\partial t}$$

A18. Characteristic equations of compressible flow (friction, heat transfer, and area effects):

$$\frac{du}{u} = \frac{1}{1 - M^2}\left(\frac{d\tau_f}{p} + \frac{dq}{c_p T} - \frac{dA}{A}\right)$$

A19. Energy equation of isentropic one-dimensional compressible flow for ideal gas:

$$\frac{p}{\rho^\gamma} = \text{const.}; \; \frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{\rho}{\rho_0}\right)^{\gamma-1} = \left[1 + \left(\frac{\gamma-1}{2}\right)M^2\right]^{-1}$$

A20. Normal shock wave of compressible flow:

$${M_2}^2 = \frac{(\gamma-1){M_1}^2 + 2}{2\gamma {M_1}^2 - \gamma + 1}$$

A21. One-dimensional compressible flow for change of area (adiabatic and frictionless):

$$\frac{A}{A_*} = \frac{1}{M}\left[\frac{2 + (\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

A22. One-dimensional Fanno flow (constant area and adiabatic):

$$\frac{\bar{f}(l_* - l)}{D} = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\ln\left[\frac{(\gamma+1)M^2}{2 + (\gamma-1)M^2}\right]$$

A23. One-dimensional Rayleigh flow (constant area and frictionless):

$$\frac{T_0}{T_{0*}} = \frac{2(\gamma+1)M^2 + (\gamma^2-1)M^4}{\gamma^2 M^4 + 2\gamma M^2 + 1}$$

A24. One-dimensional pressure wave propagation in an elastic pipe:

$$a' = \sqrt{\frac{K'}{\rho}} \; ; \; \frac{1}{K'} = \frac{1}{K} + \frac{D}{bE}$$

A25. Vector Identities:

$\nabla \cdot (\nabla \times \mathbf{a}) = 0$ $\quad$ $\nabla \times \nabla\phi = 0$ $\quad$ $\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - (\nabla \cdot \nabla)\mathbf{a}$

$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$ $\quad$ $(\mathbf{a} \cdot \nabla)\mathbf{a} = \nabla(\mathbf{a} \cdot \mathbf{a})/2 - \mathbf{a} \times (\nabla \times \mathbf{a})$ $\quad$ $\nabla \cdot (\phi\mathbf{a}) = \phi\nabla \cdot \mathbf{a} + \mathbf{a} \cdot \nabla\phi$

$\nabla \times (\mathbf{a} \times \mathbf{b}) = (\nabla \cdot \mathbf{b})\mathbf{a} - (\nabla \cdot \mathbf{a})\mathbf{b} - (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a}$ $\quad$ $\nabla \times (\phi\mathbf{a}) = \nabla\phi \times \mathbf{a} + \nabla\phi(\nabla \times \mathbf{a})$

A26. Gauss (Divergence) and Stokes (Curl) theorem :

$$\oint_A \mathbf{a} \cdot \hat{\mathbf{A}} dA = \int_V (\nabla \cdot \mathbf{a}) dV \; ; \; \oint_L \mathbf{a} \cdot \hat{\mathbf{L}} dL = \int_A (\nabla \times \mathbf{a}) \cdot \hat{\mathbf{A}} dA$$

A27. Leibnitz Rule:

$$\frac{\partial}{\partial x}\left[\int_{\alpha(x)}^{\beta(x)} \phi(x,y)\, dy\right] = \int_{\alpha(x)}^{\beta(x)} \frac{\partial \phi}{\partial x}(x,y)\, dy + \phi(x,\beta)\beta'(x) - \phi(x,\alpha)\alpha'(x)$$

End of Paper